Random Fields in Continuum Mechanics

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Classical deterministic models of continuum mechanics expressed by boundary value problems for the system of partial differential equations may fail for various reasons, especially in multiscale problems. From the standpoint of micromechanics, stochastic models such as stochastic partial differential equations (SPDE) and stochastic finite elements (SFE) naturally involve tensor-valued random fields (TRF) with generally anisotropic realizations [1] and non-trivial correlation functions [2]. By contrast, the commonly employed SPDE and SFE models employ scalar-valued random fields to represent tensor-type material properties (conductivity, piezoelectricity, elasticity…) or simply postulate their TRF with white-noise correlations.

First, we give explicit representations of most general correlation functions of TRF of 1st, 2nd, 3rd, and 4th ranks. Next, we examine the consequences dictated by field equations on dependent TRF (displacement, velocity, strain, stress…) of various ranks (2d or 3d) in classical continua and, similarly, for temperature and heat flux in conductivity. Related topics include scaling of constitutive responses in random media and waves on random fields.

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2. A. Malyarenko and M. Ostoja-Starzewski, *Tensor-Valued Random Fields for Continuum Physics*, Cambridge University Press, 2019.